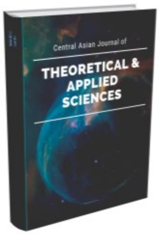

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Article

Intuitionistic Fuzzy Almost π Generalized Semi Open Mapping In Gradation Ideal Topological Spaces

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Abstract: The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy ideal almost π generalized semi open mappings and intuitionistic fuzzy ideal almost π generalized semi closed mappings in intuitionistic fuzzy ideal space and we investigate some of its properties. Also we provide the relations between intuitionistic fuzzy ideal almost π generalized semi closed mappings and other intuitionistic fuzzy ideal closed mappings .

Keywords : intuitionistic fuzzy topology, intuitionistic fuzzy ideal topology , intuitionistic fuzzy ideal π generalized Semi closed set , intuitionistic fuzzy ideal almost π generalized semi closed mappings ,Intuitionistic fuzzy ideal almost π generalized semi open mappings and intuitionistic fuzzy ideal π $T_{1/2}$ (IFI $\pi T_{1/2}$) space and intuitionistic fuzzy ideal π $g_{1/2}$(IFI $\pi g_{1/2}$) space .

Chapter One/Methodological framework

First: Research problem:

The concept of fuzzy set was firstly introduced by L.Zadeh in 1965[19] as extension of the Classical notion of set. After three years C.L.Chang in 1968 [3] , axiomatized a collection τ Of fuzzy subset of non-empty set X .Atanassov introduced the notion of intuitionistic fuzzy sets 1986 .T.R,Hamlentt[6] investigated further properties of ideal topological space and proved some results about them .The notion of intuitionistic fuzzy ideal which is consideredas a generalization of fuzzy ideals introduced and studied by A.A.Salman and S.A.Alblowi in 2012 [7]
And in 1997 D.Coker [4] gave the basic definition of intuitionistic fuzzy topological spaces .Continuing the work done in the [13], [14], [15], [15], [16], [17] ,we define the notion of Intuitionistic fuzzy almost π generalized semi closed mappings and intuitionistic fuzzy almost πgeneralized semi open mappings .We discuss characterization of intuitionistic

fuzzy ideal almost π generalized semi closed mappings and open mappings .We also established their properties and relationship with other classes of early defined forms of intuitionistic fuzzy ideal closed mappings .

Preliminaries

Definition 2.1[1] Let X be a non-empty set .An intuitionistic fuzzy set (IFS in short) A is a subset Of X characterized by membership function $\mu_A: X \to [0,1]$ and a non- membership function $v_A: X \to [0,1]$, that they associate with each point $x \in X$ it is membership grade $\mu_A(x)$ and its N0n-membership grade $v_A(x)$ such that $0 \le \mu_A(x) + v_A(x) \le 1$, that is

$$A = \{< x, \mu_A(x)\ , v_A(x), x \in X\ \}.$$

Definition 2.2 [1] Let A and B be IFSs of from $A = \{< x, \mu_A(x)\ , v_A(x) >, x \in X\ \}$ and $B = \{< x, \mu_B(x)\ , v_B(x) >, x \in X\ \}$ then

1) $A \subseteq B\ If\ and\ only\ if\ \mu_A(x) \le \mu_B(x)\ and\ , v_A(x) \ge v_B(x) forall\ x \in X.$

2) $A = B\ if\ and\ only\ if\ A \subseteq B\ and\ B \subseteq A.$

3) $A^c = \{< x, v_A(X), \mu_A(X) >\backslash x \in X\}.$

4) $A \cap B = \{< x, \mu_A(x) \cap \mu_A(x), v_A(x) \cap v_B(x) >/ x \in X\}.$

5)) $A \cup B = \{< x, \mu_A(x) \cup \mu_A(x), v_A(x) \cup v_B(x) >/ x \in X\}.$

Definition 2.3[3] An intuitionistic fuzzy topology(IFT in short) on X is a family τ of IFSs in X Satisfying the following axioms :

1) $0_\sim, 1_\sim \in \tau.$

2) $M_1 \cap M_2 \in \tau for\ any\ M_1, M_2 \in \tau.$

3) $\cup M_i \in \tau for\ any\ family\ \{M_i/i \in J\}.$

$the\ pair\ (x, \tau) is\ called\ an\ intuitionistic\ fuzzy\ topoplogical\ space (IFTS\ in\ short).$

Definition2.4[3]Let$(X, \tau) be\ an\ IFTS\ and\ A =< x, \mu_A, v_A >\ be\ an\ IFS\ in\ X\ Then\ the\ intuitionistic\ fuzzy\ interior\ and\ intuitionistic\ fuzzy\ closure\ are\ defined\ by$

$int(A) = \cup \{G/G\ is\ an\ IFOS\ in\ X\ and\ G \subseteq A\},$

$cl(A) = \cap \{K/K\ is\ an\ IFCS\ in\ X\ and\ A \subseteq K\}.$

Definition2.5[10] A subset of A of a space (X, τ) is called :

1) reguler open if $\mathrm{A} = \mathrm{int}(\mathrm{cl}(\mathrm{A})).$

2) π open if A is the union of reguler open sets .

Defuiniton2.6[10] An $IFS\ A =< x, \mu_A, v_A > in\ an\ IFTS\ (X, \tau) is\ said\ ti\ be\ an$

1) $intuitonistic\ fuzzy\ semi\ open\ set\ (IFSOS\ in\ short) if\ A \subseteq cl\big(int\ (A)\big),$

2) $intuitionistic\ fuzzy\ \alpha - open\ set\ (if\alpha OS\ in\ short) if\ A \subseteq int(cl\big(int(A)\big),$

3) $intuitionistic\ fuzzy\ reguler\ open\ set\ (IFROS\ in\ short\ ) if\ A = int\big(cl(A)\big),$

4)$intuitionistic\ fuzzy\ pre\ open\ set\ (IFPOSon\ short\ ) if\ A \subseteq int\big(cl(A)\big),$

5)$intuitionistic\ fuzzy\ semi - pre\ open\ set (IFSPOS) if\ there\ exists\ B \in IFPO(X) such\ that$

$$B \subseteq A \subseteq Cl(B).$$

Definition 2.7[10] An IFS A $=< x, \mu_A, v_A > in\ an\ IFTS\ (X, \tau) is\ said\ ti\ be\ an$

1) $intuitonistic\ fuzzy\ semi\ closed\ set\ (IFScS\ in\ short) if\ int\big(cl\ (A)\big) \subseteq A,$

2) $intuitionistic\ fuzzy\ \alpha - closed\ set\ (if\alpha cS\ in\ short) if\ int(cl\big(int(A)\big) \subseteq A,$

3) $intuitionistic\ fuzzy\ reguler\ closed\ set\ (IFRcS\ in\ short\ ) if\ A = cl\big(int(A)\big),$

4)$intuitionistic\ fuzzy\ pre\ open\ set\ (IFPOSon\ short\ ) if\ cl\big(int(A)\big) \subseteq A,$

$\boldsymbol{Defintion}\ \mathbf{2.8}[\mathbf{10}]$ An ifs A in an IFTS(X, τ)$is\ said\ to\ be\ intuitionistic\ fuzzy\ \pi\ generalized\ semi\ closed\ set\ (IF\pi GSCS\ in\ short\ ) if\ scl\ (A) \subseteq U\ whenever\ A \subseteq U\ and\ U\ is\ an\ IF\pi OS\ in\ (X, \tau). An\ IFS\ A\ is\ said\ to\ be\ an\ intuitionistic\ fuzzy\ \pi\ generalized\ semi\ open\ set\ (IF\pi GSOS\ in\ short)\ in\ X\ if\ the\ complement\ A^c\ is\ an\ IF\pi GSCS\ in\ X$.

$\boldsymbol{Definition}\ \mathbf{2.11}$[7] $Let\ f\ be\ a\ mapping\ from\ an\ IFTS\ (X, T)\ into\ an\ IFTS\ (Y, \sigma). Then\ f\ is\ said\ to\ be\ intuitionistic\ fuzzy\ continuous\ (IFcontinuous\ ) if\ f^{-1}(B) \in IFO(X) for\ every\ B \in \sigma$.

$Defintion$ 2.12 [12]$Let\ f\ be\ a\ mapping\ from\ an\ IFTS\ (X, \tau) into\ an\ IFTS\ (Y, \sigma). Then\ f\ is\ said\ to\ be\ an\ intuitionistic\ fuzzy\ generalized\ continuous\ (IFG\ \ continuous\ ) if\ f^{-1}(B) \epsilon IFGCS(X)\ for\ every\ IFCS\ B\ in\ Y$.

$\boldsymbol{Definition}\ \mathbf{2\ .13}$ [14]$Letf\ be\ a\ mapping\ from\ an\ LFTS\ (X, \tau)\ into\ an\ IFTS(Y, \sigma). Then\ f\ is\ said\ tobe\ an\ intuitionistic\ fuzzy\ almost\ \pi\ generalized\ semi\ continuous\ mappings\ \binom{IFA\ \pi GA}{continuous} if\ f^{-1}(B)\ \epsilon\ IFGCS(X) for\ every\ \ IFFCS\ Bin\ Y.$

$\boldsymbol{Defintion}\ \mathbf{2.14}$ [15] $Let\ f\ be\ a\ mapping\ from\ an\ IFTS(X, \tau) into\ an\ IFTS\ (Y, \sigma). Then\ f\ is\ said\ to\ be\ an\ intuitionistic\ fuzzy\ \ \alpha\ generalized\ contuiuous\ mappings (IF\alpha G\ continuous\ if\ f^{-1}(B)$

$\epsilon IFT\alpha GCS(X)$ $for\ every\ IFRCS\ B\ in\ Y$.

***Defintion* 2.15** [15] *Let f be a mapping from an IFTS*(X,τ)*into an IFTS* (Y,σ)*.Then f is said to be an intuitionistic fuzzy α generalized semi closed mappings(IFGSCM) if* $f^{-1}(B)$ ϵ *IFGCS*(X) *for every IFRCS B in Y* .

***Defintion* 2.16** *Let f be a mapping from an IFTS*(X,τ)*into an IFTS* (Y,σ)*.Then f is said to be an intuitionistic fuzzy almost closed mappings(IFACM) if* $f^{-1}(B)$ ϵ *IFC*(Y) *for every IFRCS B in X* .

***Defintion* 2.17** *Let f be a mapping from an IFTS*(X,τ)*into an IFTS* (Y,σ)*.Then f is said to be an intuitionistic fuzzy almost α generalized closed mappings(IFAαGCM) if* $f^{-1}(B)$ $\epsilon IF\alpha GC(Y)$ *for every IFRCS B in Y* .

Defintion 2.18 [5] *The IFS* $c(\alpha,\beta) = \langle x, c_{\alpha}, c_{1-\beta}\rangle$ *where* $\alpha\epsilon\ (0,1], \beta\ \epsilon[0,1)$ *and* $\alpha+\beta\leq 1$ *is called an intuitionistic fuzzy point (IFP)in X.*

Note that an IFP$c(\alpha,\beta)$ *is said to belong to an IFS A* $= \langle x, \mu_A, v_A\rangle$ *of X denoted by* $c(\alpha,\beta)$ ϵA *if* $\alpha\leq\mu_A$ *and* $\beta\geq v_A$.

***Defintion* 2.18 [5]** *Let* $c(\alpha,\beta)$*be an IFP of an IFTS* (X,τ)*.An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN)of*$c(\alpha,\beta)$ *if there exists an IFOS B in X such that* $c(\alpha,\beta)\in B\subseteq A$.

Defintion* 2.19** [7] ***An IFS A *is said to be an intuitionistic fuzzy dense (IFS for short) in another IFS B in an IFTS*(X,τ)*, if* $cl(A)=B$.

***Defintion* 2.20 [11]** *An IFTS* (X,τ)*is said to be an intuitionistic fuzzy* $\pi T_{1\backslash 2}$ $(IF\pi T_{1\backslash 2}$ *in short) space if every IFπGSCS in X is an IFCS IN X* .

***Defintion* 2.21 [11]** *An IFTS* (X,τ)*is said to be an intuitionistic fuzzy* $\pi T_{1\backslash 2}$ $(IF\pi gT_{1\backslash 2}$ *in short) space if every IFπGSCS in X is an IFGCS inX* .

***Result* 2.22. [9]** (1)*every IFπOS is an IFOS in* (X,τ).

(2)*every IFπCS is an IFCS in* (X,τ).

***Dfinition* 2.23 [12]** *A non empty collection of fuzzy set I of set X satisfying the condition*

1) $if\ A \in I\ and\ B \leq A, then\ B \in I\ (heredity)$,

2) $if\ A\ \in I\ and\ B \in I\ then\ A \vee B \in I(\ finite\ additivity) is\ called\ a\ fuzzy\ ideal\ on\ X.$

$The\ triplex\ (X, \tau, I)\ denotes\ a\ fuzzy\ ideal\ topological\ space\ with\ a\ fuzz\ ideal\ I\ and\ fuzzy\ topology\ \tau$.

3 − Intuotonostic Fuzzy Ideal almost π Generalized Semi Open Mappings in Gradation Topologicals Space .

In this section we introduse intuitionistic fuzzy ideal almost πgeneralized semi open mappings , intuitionistic fuzzy ideal almost πgeneralized semi closed mappings and studied some of its properties .

Defintion 3.1 A mapping $F: X \to Y$ is called an intuitionistic fuzzy ideal almost π generalized semiopen mappings (IFIAπGSOM for short)if $f(A)$is an IFIπGSOS in Y for each IFIROS A in X .

Definition 3.2 A mapping $f: (X, \tau, I) \to (Y, \sigma, I)$is called an intuitionistic fuzzy ideal almost πgeneralized semi closed mappings (IFIAπGSCM for short)if $f(B)$is an IFIπGSCS in $(Y, \sigma, I)$for each IFIRCS B in(X, τ, I) .

Defintion 3.3 Let $X = \{a, b\}, Y = \{u, v\}$and $G_1 =< x, (0.2_a, 0,2_b), (0.6_a, 0.7_b) >, G_2 =< y, (0.4_u, 0,2_v), (0.6_u, 0.7_v) >$. Then ,$\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFITs on X and Y respectively . Define a mapping $f: (X, \tau, I) \to (Y, \sigma, I)$ by $f(a) = u$ and $f(b) = v$. Then $f$is an IFIAπGSCM .

Theorem 3.4 (1)Every IFICM is an IFIAπGACM but not conversely .

(2)Every IFIαGCM is an IFIAπGSCM but not conversely .

(3)Every IFICM is an IFIAπGSCM but not conversely .

(4)Every IFIAαGCM is an IFIAπGSCM but not conversely .

proof: (**1**)Assume that$f: (X, \tau, I) \to (Y, \sigma, I)$is an IFICM. Let A be an IFIFCS in X. This implie sA is an IFICS in X. Since f is an IFICM, $f(A)$is an IFICS in Y . Every IFICS is an IFIπGSCS in Y Hence f is an IFIAπGSCM .

$\boldsymbol{proof}$: (**2**) Let $f:(X,\tau,I) \to (Y,\sigma,I)$ is an $IFI\alpha GCM$. Let A is an $IFIRCS$ in X. This implies A is an $IFICS$ in X. Then by hypothesis $f(A)$ is an $IFI\alpha GCS$ in Y. Since every $IFI\alpha GCS$ is an and every $IFIGSCS$ is an $IFI\pi GSCS$, $f(A)$ is an $IFI\pi GSCS$ in Y. Hence f is an $IFIA\pi GSCM$.

$\boldsymbol{proof}$: (**3**) Let $f:(X,\tau,I) \to (Y,\sigma,I)$ is an $IFIACM$. Let A be an $IFIRCS$ in X. Since f is $IFIACM$ $f(A)$ is an $IFICS$ in Y. Since every $IFICS$ is an $IFI\pi GSCS$, $f(A)$ is an $IFI\pi GSCS$ in Y. Hence f is an $IFIA\pi GSCM$.

$\boldsymbol{proof}$: (**4**) Let $f:(X,\tau,I) \to (Y,\sigma,I)$ is an $IFIA\alpha GCM$. Let A be an $IFIRCS$ in X. Since f is $IFIACM$, Then by hypothesis $f(A)$ is an $IFI\alpha GCS$ in Y. Since every $IFIGCS$ is an $IFIGSCS$ and every $IFIGSCS$ is an $IFI\pi GCS$, $f(A)$ is an $IFI\pi GSCS$ in Y. Hence f is an $IFIA\pi GSCM$.

$\boldsymbol{Example}$(**1**) Let $X = \{a,b\}, Y = \{u,v\}$ and $G_1 = \{x,(0.4_a,0.2_b),(0.5_a,0.4_b)\}, G_2 = \{y,(0.3_u,0.2_v),(0.6_u,0.7_v)\}$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are $IFITs$ on X and Y. Define a mapping $f:(X,\tau,I) \to (Y,\sigma,I)$ by $f(a) = u$ and $f(b) = v$. Then, f is an $IFIA\pi GS$ CM. But f is not an $IFICM$ since $G_1^c = \{x,(0,0.5_a,0.4_b),(0.4_a,0.2_b)\}$, is an $IFICS$ in X but f $(G_1^c)\{y,(0,0.5_u,0.4_v),(0.4_u,0.2_v)\}$ is not an $IFICS$ in Y.

$\boldsymbol{Example}$(**2**) Let $X = \{a,b\}, Y = \{u,v\}$ and $G_1 = \{x,(0.3_a,0.4_b),(0.4_a,0.5_b)\}, G_2 = \{y,(,0.7_u,0.6_v),(0.3_u,0.4_v)\}$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are $IFITs$ on X and Y. Define a mapping $f:(X,\tau,I) \to (Y,\sigma,I)$ by $f(a) = u$ and $f(b) = v$. Then, f is an $IFIA\pi GS$ CM. But f is not an $IFI\alpha GCM$ since $G_1^c = \{x,(0.4_a,0.5_b),(0.3_a,0.4_b)\}$, is an $IFICS$ in X but f $(G_1^c) = \{y,(0.4_u,0.2_v),(0.3_u,0.4_v)\}$ is not an $IFI\alpha GCS$ in Y.

$\boldsymbol{Example}$(**3**) In example (1), f is an $IFIA\pi GSCM$ but f is not an $IFIA\alpha GCM$ since $G_1^c = \{x,(0.4_a,0.5_b),(0.3_a,0.4_b)\}$, is an $IFIRCS$ in X but $f(G_1^c) = \{y,(0.5_u,0.4_v),(0.4_u,0.2_v)\}$ is not an $IFICS$ in Y.

$\boldsymbol{Example}$(**4**) In example (2), f is an $IFIA\pi GSCM$. But f not an $IFIA\delta GCM$ since $G_1^c =$

$\{x,(0.4_a,0.5_b),(0.3_a,0.4_b)\}$ *is an IFIRCS in* Y *but* $f(G_1^c) = \{y,(0.4_u,0.2_v),(0.3_u,0.4_v)\}$ *is not an IFIαGCS in* Y.

Theorem 3.5 *A bijective mapping* $f: X \to Y$ *is an IFIAπGS closed mapping if and only if the imge of each IFIROS in* X *is an IFIπGSOS in* Y.

Proof Necessity: *Let* A *be an IFIROS in* x. *This implies* A^c *is IFIRCS in* X. *Since* f *is an IFIAπGS closed mapping*, $f(A^c)$ *is an IFIπGSCS in* Y. *Sincs* $f(A^c) = (f(A))^c$, $f(A)$ *is an IFIπGSOS in* Y.

Sufficiency: *Let* A *be an IFIFCS in* X. *This implies* A^c *is an IFIROS in* X. *By hypothesis*, $f(A^c)$ *is an IFIπGSOS in* Y. *Since* $f(A^c) = (f(A))^c$, $f(A)$ *is an IFIπGSCS in* Y. *Hence* f *is an IFIAπGS closed mapping*.

Theprem 3.6 *Let* $f: (X,\tau,I) \to (Y,\sigma,I)$ *be an IFIAπGS closed mapping. Then* f *is an IFIA closed mapping if* Y *is an* $IFIT_{1\backslash 2}$ *space*.

Proof: *Let* A *be an IFIRCS in* X. *Then* $f(A)$ *is an IFIπGSCS in* Y, *by hypothesis. Since* Y *is an* $IFIT_{1\backslash 2}$ *space*, $f(A)$ *is an IFICS in* Y. *Hencs* f *is an IFIA closed mapping*.

Theprem 3.7 *Let* $f: X \to Y$ *be a bijective mapping. Then the following are equivalent.*

1) f *is an IFIAπGSOM*

2) f *is an IFIAπGSCM*.

proof: *Straightforward*.

Theorem 3.8 *Let*: $X \to Y$ *be mapping where* Y *is an* $IFI\pi T_{1\backslash 2}$ *space. Then the following are equivalent*:

(1) f *is an IFIAπGSCM*

(2) $scl(f(A)) \subseteq f(cl(A))$ *forevery IFISPOS* A *in* X

(3) $scl(f(A)) \subseteq f(cl(A))$ *forevery IFISOS* A *in* X.

proof (1) $\Rightarrow$ (2) *Let* A *be an IFISPOS in* X. *Then* $cl(A)$ *is an IFIRCS in* X. *By hypothesis*, $f(cl(A))$ *is an IFIπGSCS in* Y. *Since* Y *is an* $IFI\pi T_{1\backslash 2}$ *space. This implies* $scl(f(A)) =$

$f(cl(A)). Now\ scl(f(A)) \subseteq scl\left(f(cl(A))\right) = f(cl(A)). Thus\ scl(f(A))$
$\subseteq f(cl(A)).$

$(2) \Rightarrow (3) Sinceevery\ IFISOS\ is\ an\ IFISPOS, the\ proof\ directly\ follows.$

$(3) \Rightarrow (1) LetA\ be\ an\ IFIRCS\ in\ X. Then\ A$
$= cl(int(A)). Therefore\ A\ is\ an\ IFISOS\ in\ X\ . By$

$hypothesis, scl(f(A)) \subseteq f(cl(A)) = f(A)$
$\subseteq scl(f(A)). Hence\ f(A) is\ an\ IFISCS\ and$

$hence\ is\ an\ IFI\pi GSCS\ in\ Y\ . Thus\ f\ is\ an\ IFIA\pi GSCM.$

$\boldsymbol{Theorem\ 3.9}\ Let : X$
$\rightarrow Y\ be\ mapping\ \ where\ Y\ is\ an\ IFI\pi T_{1\backslash 2}\ space\ . Then\ the\ following$

$are\ equivalent:$

$(1) fis\ an\ IFIA\pi GSCM$

$(2) f(A) \subseteq sint \subseteq \left(f\left(int(cl(A))\right)\right) forevery\ IFIPOS\ A\ in\ X\ .$

$proof\ (1) \Rightarrow (2) Let\ A\ be\ an\ IFIPOS\ in\ X\ . Then\ A$
$\subseteq int(cl\ (A)). Since\ int(cl(A)) is\ an$

$IFIROS\ in\ X\ , by\ hypothesis, f(int(cl(A)) is\ an\ IFI\pi GSOS\ in\ Y\ . Since\ Y\ is\ an\ IFI\pi T_{1\backslash 2} space,$

$f\left(int(cl\ (A))\right) is\ an\ IFISOS\ in\ Y\ . Therefore\ f(A) \subseteq f\ \left(int(cl(A))\right)$
$\subseteq sint\left(f\left(int(cl(A))\right)\right)$

(2)
$\Rightarrow (1) Let\ A\ be\ an\ IFIROS\ in\ X\ . Then\ A\ is\ an\ IFIPOS\ in\ X\ . By\ hypothesis\ , f(A)$
$\subseteq sint(f$

$(cl(A)))) = sint\ (f(A))$
$\subseteq f(A). This\ implies\ f(A)\ is\ an\ IFISOS\ in\ Y\ and\ hence\ is\ an\ IFI\pi G$

$SOS\ \ in\ Y\ . Thereforef\ is\ an\ IFIA\pi GSCM, by\ theorem\ 3.6\ .$

$\boldsymbol{Theorem\ 3.10}\ Let : (X, \tau, I)$
$\rightarrow (Y, \sigma, I) be\ mapping\ froman\ IFITS\ X\ into\ an\ IFITS\ Y.$

$Then\ the\ following\ conditions\ are\ equivalent\ if\ Y\ is\ an\ IFI\pi T_{1\backslash 2}\ space.$

$(1) fis\ an\ IFIA\pi GSCM$

$(2) f\ is\ an\ IFIA\pi GSOM$

$(3) f(int(A)) \subseteq int\left(cl\left(int(f(A))\right)\right) forevery\ IFIROS\ A\ in\ X\ .$

$proof(1) \Rightarrow (2) It\ is\ obviously\ true.$

$(2) \Rightarrow (3) LetA\ be\ any\ IFIROS\ in\ X\ . This\ implies\ A\ is\ an\ IFIOS\ in\ X. Then\ int(A) is\ an\ IFIOS$

$in\ X\ .Then\ f\big(int(A)\big) is\ an\ IFI\pi GSOS\ in\ Y. Since\ is\ an\ IFI\pi T_{1\backslash 2} space, f\big(int(A)\big) is\ an\ IFIOS\ in$

$Y\ .Therefore\ f\big(int(A)\big) = int(f\big(int(A)\big) \subseteq int\Big(cl\Big(int\big(f(A)\big)\Big)\Big).(3)$
$\Rightarrow (4) Let\ A\ be\ an$

$IFIRCS\ in\ X. Then\ itscomplement\ A^c is\ an\ IFIROS\ in\ X. By\ hypothesis\ f(int(A^c)) \subseteq int(cl$

$(int\big(f(A^c)\big)))). This\ implies\ f(A^c)$
$\subseteq int(cl(int(f(A^c)))). Hence\ f(A^c) is\ an\ IFI\alpha OS\ in\ Y.$

$Sinc\ every\ IFI\alpha OS\ is\ an\ IFI\pi GSOS, f(A^c) is\ an\ IFI\pi GSOS\ in\ Y. Therefore f(A) is\ an\ IFI\pi GSCS$

$in\ Y. Hence\ f\ is\ an\ IFIA\pi GSCM\ .$

$\boldsymbol{Theorem\ 3.11}\ Let\ : (X, \tau, I)$
$\rightarrow (Y, \sigma, I) be\ mapping\ froman\ IFITS\ X\ into\ an\ IFITS\ Y.$

$Then\ the\ following\ conditions\ are\ equivalent\ if\ Y\ is\ an\ IFI\pi T_{1\backslash 2}\ space.$

$(1) f is\ an\ IFIA\pi GSCM$

$(2) scl) f(A)) \subseteq f(cl(A))\ for\ every\ IFISCS\ in\ X.$

$proof (1) \Rightarrow (2) Assume\ that\ A\ is\ an\ IFIRCS\ in\ X. By\ Definition, int\big(cl(A)\big) \subseteq A.$

$This\ implies\ cl(A) is\ an\ IFIRCS\ in\ X. By\ hypothesis\ f\big(cl(A)\big)\ is\ an\ IFI\pi GSCS\ in\ Y\ and$

$hence\ is\ an\ IFI\pi CS\ in\ Y, since\ Y\ is\ an\ IFI\pi T_{1\backslash 2} space. This\ implies\ scl(f\big(cl(A)\big)$
$= f\big(cl(A)\big)$

$= f\big(cl(A)\big). Now\ scl(f(A)) \subseteq scl(f\big(cl(A)\big)$
$= f\big(cl(A)\big). Since\ cl(A) is\ an\ IFIROS, int(cl$

$\big(cl(A)\big)) = cl(A). This\ implies\ scl\ \big(f(A)\big) = f(int\ \Big(cl\big(cl(A)\big)\Big))$
$\subseteq f(A \cup int\big(cl(A)\big))) = f(scl$

$(A). Hence\ scl\ \big(f(A)\big) \subseteq f\big(scl(A)\big). (2) \Rightarrow (1) Let\ A\ be\ an\ IFIRCS\ in\ X. Then\ A$
$= cl\big(int(A)\big).$

$Therefore\ A\ is\ an\ IFISCS\ in\ X\ . By\ hypothesis, scl\big(f(A)\big) \subseteq f\big(scl(A)\big)$
$\subseteq f\big(cl(A)\big) = f(A)$

$\subseteq scl\big(f(A)\big). That\ is\ scl\big(f(A)\big)$
$= f(A). Hence f(A) is\ an\ IFI\pi CS\ \ and\ hence\ is\ an\ IFI\pi GSCS\ in$

$Y. Thus\ f\ is\ an\ IFIA\pi GSCM.$

$\boldsymbol{Theorem\ 3.12}\ Let\ : (X, \tau, I)$
$\rightarrow (Y, \sigma, I) be\ mapping\ froman\ IFITS\ X\ into\ an\ IFITS\ Y.$

$Then\ the\ following\ conditions\ are\ equivalent\ if\ Y\ is\ an\ IFI\pi T_{1\backslash 2}\ space.$

$(1) f is\ an\ IFIA\pi GSCM$

$(2) f(A) \subseteq \pi int\ \Big(f\big(scl(A)\big)\Big) for\ every\ IFIPOS\ A\ in\ X.$

$proof: (1) \Rightarrow (2)$ $Let\ A\ be\ an\ IFIPOS\ in\ X. Then\ A \subseteq int(cl(A)). Since\ int(cl(A)) is\ an\ IFIROS$

$in\ X\ . by\ hypothesis, f(int(cl(A)) is\ an\ IFI\pi GSOS\ in\ Y. Since\ Y\ is\ an\ IFI\pi T_{1/2}\ space, f(int(cl(A))$

$is\ an\ IFI\pi OS\ in\ Y. Therefore\ f(A) \subseteq f\left(int(cl(A))\right) \subseteq \pi\ int(f(int(cl(A))) \subseteq \pi int$

$(f\left(A \cup int(cl(A))\right) = \pi int(f\left(scl((A))\right). That\ is\ f(A) \subseteq\ \pi int\left(f(scl(A))\right).$ $(2) \Rightarrow (1)$ $Let\ A$

$be\ an\ IFIROS\ in\ X. Then\ A\ is\ an\ IFIPOS\ in\ X\ . By\ hypothesis, f(A) \pi int\left(f(scl(A))\right). This\ implies$

$f(A) \subseteq \pi int\left(f\left(A \cup int(cl(A))\right)\right) \subseteq \pi int(f(A \cup A)) \pi int(f(A)) \subseteq f(A). Therefore\ f(A) is\ an$

$IFI\pi OS\ in\ Y\ and\ hence\ an\ IFI\pi GOS\ in\ Y. Thus\ f\ is\ an\ IFIA\pi GS\ closed\ mapping\ .$

Theorem 3.13 $Let : (X, \tau, I) \rightarrow (Y, \sigma, I) be\ a\ mapping\ from\ an\ IFITS\ X\ into\ an\ IFITS\ Y.$

$Then\ the\ following\ conditions\ are\ equivalent\ if\ Y\ is\ an\ IFI\pi T_{1\backslash 2}\ space.$

$(1) f is\ an\ IFIA\pi GSCM$

$(2) If\ B\ is\ an\ IFIROS\ in\ X\ then\ f(B) is\ an\ IFI\pi GSOS\ in\ Y$

$(3) f(B) \subseteq int(cl(f(B)) for\ every\ IFIROS\ in\ X\ .$

$Proof\ (1) \Rightarrow (2) obviously.$

$(2) \Rightarrow (3) Let\ B\ be\ any\ IFIROS\ in\ X. Then\ by\ hypothesis\ f(B) is\ an\ IFI\pi GSOS\ \ in\ Y. Since\ X\ is\ an$

$IFI\pi T_{1/2}\ \ space, f(B) is\ an\ IFIOS\ in\ Y\ (Result\ 2.23\ ). Therefore\ f(B) = int(f(B)) \subseteq int$

$\left(cl(f(B))\right).$ $(3) \Rightarrow (1) LetB\ be\ an\ IFIRCS\ in\ X. Then\ its\ complement\ B^c is\ an\ IFIROS\ in\ X.$

$By\ hypothesis\ f(\ B^c) \subseteq int\left(cl(f(B^c))\right). Hence\ f(B^c)\ is\ an\ IFI\pi OS\ in\ Y. Since\ every\ IFI\pi OS$

$is\ an\ IFI\pi GSOS, f(B^c) is\ an\ IFI\pi GSOS\ in\ Y. Therefore\ f(B) is\ an\ IFI\pi GSCS\ \ in\ Y. Hence\ f\ is$

$an\ IFIA\pi GSCM.$

Theorem 3.13 $Let : (X, \tau, I) \rightarrow (Y, \sigma, I) be\ a\ mapping\ from\ an\ IFITS\ X\ into\ an\ IFITS\ Y.$

$Then\ the\ following\ conditions\ are\ equivalent\ if\ Y\ is\ an\ IFI\pi T_{1\backslash 2}\ space.$

$(1) f is\ an\ IFIA\pi GSCM\ .$

$(2) int\ \left(cl(f(A))\right) \subseteq f(A) for\ every\ IFIRCS\ A\ in\ X\ .$

$Proof$ (1) $\Rightarrow$ (2) Let A be an IFIRCS in X. BY hypothesis, $f(A)$ is an $IFI\pi GSCS$ in Y. Since Y is an $IFI\pi T_{1/2}$, $f(A)$ is an IFICS in Y (Result 2.23). Therefore $cl(f(A)) = f(A)$. Now $int(cl(fA))) \subseteq cl(f(A)) \subseteq f(A)$. (2) $\Rightarrow$ (1) Let A be an IFIRCS in X. By hypothesis $int(cl(f(A))) \subseteq f(A)$. This implies $f(A)$ is an $IFI\pi CS$ in Y and hence $f(A)$ is an $IFI\pi GSCS$ in Y. Therefore f is an $IFIA\pi GSCM$.

Theorem 3.14 Let $f:(X,\tau,I) \to (y,\sigma,I)$ be an IFIA closed mapping and $g:(y,\sigma,I) \to (Z,\delta,I)$ is $IFIA\pi GS$ closed mapping, then $gof:(X,\tau,I) \to (Z,\delta,I)$ is an IFIA closed mapping. if Z is an $IFI\pi T_{1/2}$ space.

proof : Let A be an IFIRCS in X. Then $f(A)$ is an IFICS in Y. Since g is an $IFI\pi GS$closed mapping, $g(f(A))$ is an $IFI\pi GSCS$ in Z. Therefore $g(f(A))$ is an IFICS in Z, by hypothesis. Hence gof is an IFIA closed mapping.

Theorem 3.15 Let $f:(X,\tau,I) \to (y,\sigma,I)$ be an IFIA closed mapping and $g:(y,\sigma,I) \to (Z,\zeta,I)$ is $IFI\pi GS$ closed mapping, then $gof:(X,\tau,I) \to (Z,\zeta,I)$ is an $IFIA\pi GS$ closed mapping.

proof : Let A be an IFIRCS in X. Then $f(A)$ is an IFICS in Y., by hypothesis. Since g is an $IFI\pi GS$closedmapping, $g(f(A))$ is an $IFI\pi GSCS$ in Z. Therefore $g(f(A))$ is an $IFI\pi GSCS$ in Z. Hence gof is an $IFIA\pi GS$ closed mapping.

Theorem 3.16 If $f:(X,\tau,I) \to (Y,\sigma,I)$ is an $IFIA\pi GS$ closed mapping and Y is an $IFI\pi gT_{1/2}$ space, then $f(A)$ is an IFIGCS in Y for every IFIRCSA in X.

Proof: Let $f:(X,\tau,I) \to (Y,\sigma,I)$ be a mapping and let A be an IFIRCS in X. Then by hypothesis $f(A)$ is an $IFI\pi GSCS$ in Y. Since Y is an $IFI\pi T_{1/2}$ space, $f(A)$ is an IFIGCS in Y.

Theorem 3.17 Let $c(\alpha,\beta)$ be an IFIP in X. A mapping $f:X \to Y$ is an $IFI\alpha\pi GSOM$ if for every IFIOS A in X with $f^{-1}(c(\alpha,\beta)) \in A$, there exists an IFIOS B in Y with $c(\alpha,\beta) \in B$ such $f(A)$ is IFID in B.

$Prooof: Let\ A\ be\ an\ IFIROS\ in\ X.\ Then\ A\ is\ an\ IFIOS\ in\ X. Let\ f^{-1}(c(\alpha,\beta)) \in A, then\ there$

$exists\ an\ IFIOS\ B\ in\ Y\ such\ that\ c(\alpha,\beta) \in B\ and\ cl(f(A)) = B. Since\ B\ is\ an\ IFIOS, cl\ (f(A))$

$= B\ is\ also\ an\ IFIOS\ in\ Y\ . Therefore\ int\ (cl(f(A)) = cl(f(A)). Now\ f(A) \subseteq cl(f(A)) = int$

$(cl(f(A))) \subseteq cl(int(int(cl(f(A))))) = cl\left(int\left(cl(F(A))\right)\right). Thus\ f\ is\ an\ IFIA\pi GSOM.$

$Theorem$ 3.18 $Let\ f: X \to Y\ \ be\ a\ bijective\ mapping\ . Then\ the\ following\ are\ equivalent\ .$

(1)$f\ is\ IFIA\pi GSOM$

(2) $f\ is\ an\ IFIA\pi GSCM$

(3) $f^{-1}\ is\ an\ IFIA\pi GS\ \ continuous\ mapping$

$proof$ (1) $\Leftrightarrow$ (2)$isobvious\ from\ the\ theorem$ 3.7 .

(2) $\Leftrightarrow$ (3)$Let\ A \subseteq X\ be\ an\ IFIRCS. Then\ by\ hypothesis\ , f(A) is\ an\ IFI\pi GSCS\ in\ Y\ . That\ \ is$

$(f^{-1})^{-1}(A) is\ an\ IFI\pi GSCS\ in\ Y\ . This\ implies\ f^{-1}\ is\ an\ IFIA\pi GS\ continuous\ mapping\ .$

(3) $\to$ (2)$LetA \subseteq X\ be\ an\ IFIRCS. Then\ by\ hypothesis\ (f^{-1})^{-1}(A) is\ an\ IFI\pi GSCS\ in\ Y\ .$

$That\ is\ f(A) is\ an\ IFI\pi GSCS\ in\ Y. Hence\ f\ is\ an\ IFIA\pi GSCM\ .$

$Theorem$ 3.19 $Let\ f: X \to Y\ be\ a\ mapping. If\ f(sint\ (B)) \subseteq sint(f(B)) for\ every\ \ IFIS\ B\ in\ X,$

$then\ f\ is\ an\ IFIA\pi GSIM.$

$proof\ : Let\ B \subseteq X\ be\ an\ IFIROS\ . By\ hypothesis, f(sint(B)) \subseteq sint(f(B)). since\ B\ is\ an$

$IFIROS\ , it\ is\ an\ IFISPOS\ in\ X. Therefore\ sint\ (B) = B. Hence\ f(B) = f(sint(B)) \subseteq sint$

$(f(B)) \subseteq f(B). This\ implies\ f(B) is\ an\ IFISOS\ and\ hence\ an\ IFI\pi GSOS\ in\ Y. Thus\ f\ is\ an$

$IFIA\pi GSOM.$

$\boldsymbol{Theorem 3.20}$ $Let\ f: X \to Y\ be\ a\ mapping. If\ scl(f(B)) \subseteq f(scl(B)) forevery\ \ IFIS\ B\ in\ X,$

$then\ f\ is\ an\ IFIA\pi GSCM.$

$Proof: Let\ B\ \subseteq X\ be\ an\ IFIRCS. By\ hypothesis, scl(f(B)) \subseteq f(scl(B)). Since\ B\ is\ an\ IFIRCS,$

$it\ is\ an\ IFISCS\ in\ X. Therefore\ scl(B) = B. Hence\ f(B) = f(scl(B)) \supseteq scl(f(B)) \supseteq f(B).$

$This\ implies\ f(B) is\ an\ IFISCS\ and\ hence\ an\ IFI\pi GSCS\ in\ Y. Thus\ f\ is\ an\ IFIA\pi GSCM.$

***Theorem* 3.21** $Let\ f: X \to Y\ be\ a\ mapping\ where\ Y\ is\ an\ IFI\pi T_{1/2}\ space. If\ f\ is\ an\ IFIA\pi GSCM, then\ f(sint(B)) \subseteq cl(ont(cl(f(B)))\ for\ every\ IFIROS\ B\ in\ X.$

$Proof: This\ theorem\ can\ be\ easily\ proved\ by\ taking\ complement\ in\ theorem$ 3.19.

$Theorem$ 3.22 $Let\ f: X \to Y\ be\ an\ IFIA\pi GSOM, where\ Y\ is\ an\ IFI\pi T_{1/2}\ space. Then\ for\ each\ IFIP\ c(\alpha,\beta) in\ Y\ and\ each\ IFIROS\ B\ in\ X\ such\ that\ f^{-1}(c(\alpha,\beta)) \in B, cl\left(f(cl(B))\right) is\ an\ IFISN\ of\ c(\alpha,\beta) in\ Y.$

$Proof: Let\ c(\alpha,\beta) \in Y and\ let\ B\ be\ an\ IFIROS\ in\ X\ such\ that\ f^{-1}(c(\alpha,\beta)) \in B. That\ is\ c(\alpha,\beta) \in f(B). By\ hypothesis, f(B) is\ an\ IFI\pi GSOS\ in\ Y. Since\ Y\ is\ an\ IFI\pi T_{1/2}\ space, f(B) is\ an\ IFISOS\ in\ Y. Now\ c(\alpha,\beta) \in f(B) \subseteq f(cl(B)) \subseteq cl\left(f(cl(B))\right). Hence\ cl\left(f(cl(B))\right) is\ an\ IFISN\ of\ c(\alpha,\beta) in\ Y.$

***Remark* 3.23** $If\ an\ IFIS\ A\ in\ an\ IFITS\ (X,\tau) is\ an\ IFI\pi GSCS\ in\ X, then\ \pi gscl(A) = A. But\ the\ converse\ may\ not\ be\ true\ in\ general, since\ the\ intersection\ does\ not\ exist\ in\ IFI\pi GSCSs.$

***Remarek* 3.24** $If\ an\ IFIS\ A\ in\ an\ IFITS\ (X,\tau) is\ an\ IFI\pi GSOS\ in\ X, then\ \pi gsint(A) = A. But\ the\ converse\ may\ not\ be\ true\ in\ general, since\ the\ union\ foes\ not\ exist\ in\ IFI\pi GSOSs.$

***Theorem* 3.25** $Let\ f: X \to Y\ be\ a\ mapping. If\ f\ is\ an\ IFIA\pi GSCM, then\ \pi gscl(f(A)) \subseteq f(cl(A)) forevery\ IFISOS\ A\ in\ X.$

$Proof: Let\ A\ be\ an\ IFISOS\ in\ X. Then\ cl(A) is\ an\ IFIRCS\ in\ X. By\ hypothesis\ f(cl(A)) is\ an\ IFI\pi GSCS\ in\ Y. Then\ \pi gscl(f(cl(A)) = f(cl(A)). Now\ \pi gscl(f(A)) \subseteq \pi gscl\left(f(cl(A))\right) = f(cl(A)). That\ is\ \pi gscl(f(A)) \subseteq f(cl(A)).$

$Corollary$ 3.26 $Let\ f: X \to Y\ be\ a\ mapping. If\ f\ is\ an\ IFIA\pi GSCM, then\ \pi gscl(f(A)) \subseteq f(cl(A)) for\ every\ IFIGSOS\ A\ in\ X.$

$Proof$: $Since\ every\ IFISOS\ is\ an\ IFISOS\ is\ an\ IFIGSOS, the\ proof\ is\ obvious\ from\ the$

$\boldsymbol{Theorem\ 3.25}$.

REFERENCES

[1] K.Atanassov,Intuitionistic fuzzy sets,Fuzzy Sets and Systems,20(1986),87-96 .
[2] B.Krsteska and E.Ekici,Intuitionistic fuzzy contra strong ore continuity,Faculty of Sciences and Mathematics,21(2)(2007),273-284.
[3] C.Chang,Fuzzy topological spaces,J.Math.Appl.,24(1968),182-190.
[4] D.Coker,An introdiction to fuzzy topological space,Fuzzy Sets and Systems,88(1997),81-89.
[5] Coker and M.Demirci,On intuitionistic fuzzy points,Notes on IFS,(1995),79-84.
[6] T.R,Hamlett and D.Jankovic,Ideals in topological space,Int.J.contemp.Math.Sciences, Vol.(7),4-B(1990),864-874.
[7] A.A.Salama and AL.Blowi,"intuiuionistic fuzzy ideal topological space",Journal advances in fuzzy mathematics(AFM).VOLUME 7no .1(2012),pp.51-60.
[8] Y.MSaber and M.A.Abdel-sattar "Ideal On Fuzzy Topological Space" Applied Mathematical Sciences,Vol.8,(2014),no.34,1667-1691.
[9] M.E.EI-Shafhi and A.Zhakari,Semi generalized continuous mappings in fuzzy topological spaces ,J.Egypt.Math.soc.,15(1)(2007),57-67.
[10] H.Gurcay,D.Coker and A.Haydar,ON fuzzy continuity in intuitionistic fuzzy topological spaces,Jour.of Fuzzy Math.,5(1997),365-378.
[11] I.M.Hanafy,Intuitionistic fuzzy γ continuity,Canad.Math Bull.XX(2009),1-11.
[12] J.K.Jeon,Y.B.Jun and J.H.Park,Intuitionistic fuzzy alpha continuity and intuitionistic
[13] fuzzy pre continuity,Intuitionistic Journal of Mathematics and mathematical Sciences,19 (2005),3091-3101.
[14] S.Maragathavalli and K.Ramesh,Intuitionistic fuzzy π generalized semi sets,Advances in Theoretical and Applied Sciences,1(2012),33-42.
[15] S.Maragathavallia and K.Ramesh,Intuitionistic fuzzy π generalized semi irresolute mappings,International of Mathematical Archive,3(2012),1-7.
[16] S .Maragathavallia and K.Ramesh,Intuitionistic fuzzy π generalized semi continuousmappings,International Journal of Computer Applications,37(2012),30-34.
[17] S .Maragathavallia and K.Ramesh,Intuitionistic fuzzy completely π generalized semi continuous mappings ,(Submitted).
[18] S.Maragathavallia and K.Ramesh,On almost generalized semi continuous mappings in intuitionistic fuzzy topological spaces,Mathematical Theory and Modeling,2(2012),18-28 .
[19] S .Maragathavallia and K.Ramesh,π generalized semi closed mappings mappings,in intuitionistic fuzzy topological spaces,Journal of Advanced Studies in Topology3(2012),111-118.
[20] L.A.Zadeh,Fuzzy sets,Inrormation Control,8(1965),338-353.