Problems Associated with the Propagation of Solitons in Fibres

Najaa Baqer Qumer
Ministry of Education / Directorate of Education in Thi Qar Province/ Iraq
jjh71680@gmail.com

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Abstract: The modern-day technique of optical solitons seems to hold immense promise for transmitting
digital data at exceedingly high rates, with potential performance levels that go beyond those achievable
with traditional linear systems. The focus of the present study, as detailed in this paper, pertains to the
examination of soliton pulse propagation. Specifically, An optimal balance between self-phase
modulating (SPM) with the group velocity dispersal (GVD) has been found through the use of the
nonlinear Schro equation (NLSE) with the Fourier transform split-step method (SSF) in numerical
resolution., as represented by SPM + GVD = 0. This equilibrium serves to generate a soliton wave that
propagates through an optical fiber without undergoing any deformation, thereby preserving its shape
and velocity during propagation. The accuracy of this assertion was substantiated through the COMSIS
simulation of soliton propagation for two cases, namely, the fundamental and higher-order ones.

Keywords: solitons; nonlinear effects in fibers; group velocity dispersion; self-phase modulation.

1- INTRODUCTION
The issue of dispersion has been a prevalent challenge in optical fibre communication, particularly when
dealing with high bit-rates and long haul communication. However, solitons, which are optical pulses
based on the concept of maintaining their shapes over long distances of several thousands of kilometres,
have found immense applications in optical networks that are responsible for carrying massive amounts of
information. It is worth noting that a spatial soliton is formed as a result of the balance between the self-
focussing of an optical beam and its natural diffraction-induced spreading. On the other hand, temporal
solitons are formed due to the counteraction of the natural dispersion broadening of an optical pulse by
the Self Phase Modulation (SPM). Interestingly, the earliest example of temporal solitons is related to the
discovery of the self-induced transparency (SIT) in 1967 [1].

2- THE SOLITON
The soliton, which is a pulse, exhibits an exceptional characteristic of being capable of propagation
without modification over extended distances through the utilization of Effects of both linear and
nonlinear dynamics are mutually compensating. In order to produce soliton-mode pulses, the utilization of
a semiconductor laser is employed. The laser cavity is equipped with an element that introduces locking,
causing the longitudinal modes to interact in a way that locks them in time with one another. The soliton pulse is created when two modes lock together [2]. During the propagation of waves, temporal pulse broadening arises due to chromatic dispersion. This is because of group velocity dispersion (GVD), which causes the group velocity or delay to shift depending on the wavelength. Because the optical source are not uniform in colour, the following side effects must be taken into mind:

The phenomenon of dispersion of dispersive material (DM) can be observed in the variation of the index with respect to wavelength. The index, which is a measure of the refractive properties of the material, exhibits a pronounced dependence on the spectral characteristics of the incident light. This is due to the complex interplay between the material properties and the electromagnetic wave, which results in a dispersion of the refractive index. This phenomenon has been the subject of extensive research in the field of optics and has important implications for the design and optimization of optical devices. Therefore, a thorough understanding of the mechanisms underlying DM dispersion is crucial for the development of advanced optical systems and devices.

The phenomenon of dispersion of the waveguide (DG) is characterized by the variation of group velocity with respect to the wavelength, which is a well-known and widely studied phenomenon in the field of waveguide propagation. This variation is a fundamental property of waveguides and is of paramount importance in understanding the behavior of electromagnetic waves in these structures. In particular, the dispersion of the waveguide (DG) plays a crucial role in determining the spectral and temporal characteristics of the waveguide response, which in turn governs the performance of various optical devices and systems that rely on waveguide propagation. Therefore, a thorough investigation and analysis of the dispersion of the waveguide (DG) is essential for the development of advanced optical technologies and applications.

3- SELF PHASE MODULATION (SPM)

One major consequence of the non-linear optical phenomenon known as the Kerr effect, is the occurrence of self-phase modulation (SPM), as depicted in Figure 2. This phenomenon arises when a high intensity optical pulse propagates through a medium, inducing an index change that engenders parasitic phase modulation. The phase modulation generated under these conditions is attributed to the non-linear phase \( \Phi_{NL} \), which can be mathematically expressed as follows. Albeit the manifestation of this effect may seem trivial, it holds immense scientific significance and has numerous applications in various fields.:

\[
\Phi_{NL}(t) = \frac{2\pi}{\lambda} n_2 L i(t) \tag{1}
\]

in this particular context, it is of utmost importance to elucidate that the aforementioned symbol \( n_2 \) is the total length of an optical fibre and \( N \) is the nonlinear index. It is to be noted that the phase, which is contingent upon time, causes alterations in both the width as well as the shape of the spectrum. As a logical corollary, it follows that the instantaneous angular frequency can be expressed as per the equation mentioned in reference number three.

\[
\omega(t) = \omega_0 - \delta\omega(t) \tag{2}
\]

\[
\delta\omega(t) = \frac{d}{dt}\Phi_{NL}(t) \tag{3}
\]
Fig 1: Temporal dependency of the intensity shift when the instantaneous frequency is taken into account.

Fig 2: Case study of SPM's impact on a WDM optical transmission signal

The chirp is caused by the SPM phenomenon, which modifies the pulse's spectrum profile as time passes by modifying the pulse's non-linear phase and, by extension, its instantaneous frequency. Starting at the beginning of the light pulse, photons with a lower frequency than the carriers (0) are created, whereas photons with greater frequencies than m0 are produced towards the end of the light pulse. This complex modulation results in a chirp in the pulse's frequency.

\[ \partial \omega(t) = \omega(t) - \omega_0 = \frac{\partial \phi_{NL}}{\partial t} = -\gamma(\omega_0)PL \]  

(4)

Fig 3: This diagram depicts the self-phase modulation phenomena in an optical fibre, which causes a Gaussian pulse to chirp. Gaussian pulse onset time should be entered here.

4- COMPARISON BETWEEN SPM AND GVD

a. Based on the observation of the phenomenon known as LD>>LNL (local dispersion-dominated regime), one can logically infer that the system in question is predominantly dispersive in nature. When a pulse, devoid of any chirping, traverses through this medium, it is subjected to a temporal broadening effect, primarily and predominantly.

b. LD In this scenario, spectral widening of the pulse is caused by the action of SPM.
c. Recognising the importance of both dispersal & self-phase modulation (SPM) to the pulse's development as it travels, we may state categorically that the LD LNL scenario is tenable. This claim rests on the idea that there are essentially two scenarios here:

Self-phase modulation (SPM) causes the generation of long-wavelength components at the pulse's leading edge, the dispersive system can be classified as normal if and only if the dispersion parameter (D) is nonzero, which in turn causes a faster spread of these components due to the aforementioned negative dispersion. It is noteworthy that both the effects of SPM The pulse's temporal spread is helped along by both and dispersion-varying gain (DVG).

The dispersive system may be deemed abnormal when the value of D, which denotes the dispersion parameter, exceeds 0. In such a scenario, the components arising from the self-phase modulation (SPM) tend to propagate at a relatively slower pace. Consequently, there is a compensatory mechanism that takes place between the dispersion and SPM, leading to the emergence of pulse trains known as "solitons" in a passive optical fiber. This compensatory phenomenon, which occurs between the group velocity dispersion (GVD) and SPM, plays a pivotal role in the formation of such solitonic structures. The intricate interplay between these two parameters is of paramount importance in understanding the dynamics of the dispersive system.

**Fig 4:** Evolution of soliton in normal dispersion regime.

**Fig 5:** Evolution of soliton in anomalous dispersion regime.
5- SCHDINGER'S EQUATION IN A NONLINEAR FRAMEWORK

The Schrödinger equation with non-linear terms, a mathematical formulation, is commonly employed to depict and investigate the propagation of light in optical fibers. Light's physical behaviour is more accurately represented by this equation because it takes into account both nonlinear as well as linear events that occur inside the optical fibre. This crucial equation has been extensively examined and utilized in various research fields, including fiber-optic communication and nonlinear optics. The incorporation of nonlinear effects in the Schrödinger equation is particularly significant as it enables the examination of a broader range of optical phenomena, such as self-phase modulation, four-wave mixing, and stimulated Raman scattering, which cannot be elucidated solely by linear models.

While \( t \) represents time, \( Z \) represents the propagation's distance. It must be stated that \( \alpha \), on the other hand, is an attenuation parameter, while \( \beta_2 \) pertains to the chromatic dispersion term. In the endeavor of solving this particular problem, the aforementioned variables must be taken into account.

In the first case of the provided equation, it is deemed necessary to omit the dispersion part, which is commonly referred to as Group Velocity Dispersion (GVD), resulting in a value of zero. As for the second case, it is imperative to disregard the nonlinearity part, also known as the Kerr effect, which ultimately yields a value of zero.

### 5.1. A solution that is based on analysis.

#### 5.1.1. Dispersive analytical solution

\[
i \frac{\delta A(z,t)}{\delta z} = \frac{\beta_2}{2} A(z,t) \frac{\delta^2 A(z,t)}{\delta t^2} - \gamma |A(0,t)|^2 A(z,t) = 0 \quad (5)
\]

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The frequency domain form of this equation is as follows:

\[
i \frac{\delta \tilde{A}}{\delta Z} = -\frac{\beta_2}{2} w^2 \tilde{A} \quad (7)
\]

The solution is then:
\[ \hat{A}(Z,w) = \hat{A}(0,w) \exp \left( i \frac{\beta_2}{2} w^2 \right) Z \quad (8) \]

The current expression, upon close examination, clearly indicates that the ramifications of the dispersion, in their entirety, do not exert any influence whatsoever upon the spectrum under scrutiny.

### 5.1.2. Methods of non-linear analysis

In the present scenario, it can be ascertained that the dispersive effects can be conveniently disregarded in light of the nonlinear effects, as the value of \( \beta_2 \) is equal to zero or alternatively, \( D \) is equal to zero. Consequently, it can be deduced that the equation (5) is reduced to a simpler form:

\[ i \frac{\delta A}{\delta z} = \gamma |A|^2 A \quad (9) \]

The equation's solution can be expressed in the following manner:

\[ A(z, t) = A(0, T) \exp(i\gamma |A(0, t)|^2 Z \quad (10) \]

Non-linear effects refer to observed phenomena that cannot be elucidated by a linear correlation between the input and output variables, do not induce any alteration to the temporal profile of the pulse, a unit of electromagnetic radiation that propagates through space and time. However, it is worth noting that this particular effect does bring about a change in the phase of the pulse on its own, a phenomenon known as phase self-modulation. This occurrence results in a non-linear phase shift, which can be defined as a deviation from the expected phase shift. It is noteworthy to mention that this shift is often observed in the context of optics and photonics, where the study of light and its interaction with matter is of great importance [7,8].

- The Split-Step Fourier (SSF) approach for numerically solving the nonlinear Schrödinger equation
- In the first step, it is imperative that we consider solely the linear operator which is denoted by the symbol \( D \) \((N = 0)\). Calculating the operator \( D \) in the Fourier domain is necessary for simplification reasons due to the fact that In terms of spatial frequencies, the / \( t \) differential operator is equivalent to a multiplier by \( i \):

\[ A'TF^{-1} [\exp(\delta z D)TF[A(z, t)]] \quad (11) \]

\[ (z + \delta z, t) = TF^{-1} [\exp \left( i \frac{\beta_2}{2} \right) TFA(z, t)] \quad (12) \]

Through the process of iteratively applying the aforementioned steps in a significant magnitude, we are able to obtain a proximate and intimate comprehension of the progression and development of the electric field in relation to the propagation distance denoted by the variable \( z \). The subsequent depictions, illustrations, and diagrams serve to expound upon and further elaborate the intricate and nuanced procedures involved in Fourier transforms.
6- SIMULATION AND RESULTS

6.1. Transmission of a Pulse Train of Solitons via an Optical Fibre

A block schematic of the four major components of the soliton-pulse-process-based optical connection is shown below. The first component is the clock, which sends out the signal. The second component is the soliton pulse train, with the output pulse represented by the symbol (SOLT). The optical fibre is the third component, and the pulse that emerges from the fibre is symbolised by the symbol (Sf). The result is the fourth component. This research also includes a graphic depicting the power profile of a typical soliton pulse train. After travelling via a single-mode fibre, the power profile of both the a first-order (N = 3) and third-order (N = 3) soliton trains are shown side-by-side. It is possible for the fundamental a soliton pulse (N = 1, P0 = 5 mw) to propagate unaltered across an optical fibre (Fig. 9 b) while still retaining its soliton characteristics. In this work, we provide a specific example of a pulse that is a higher-order solitons, in particular one with N equal to 3 and P0 equal to 15 milliwatts. Figure 9.c depicts the generation of soliton pulsed bursts, followed by repeated spikes that increase its peak strength, illustrating the cyclical nature of this pulse's propagation. In addition, at L = LD / N metres, or 38.45 km, the pulse reverts to its original form.

Fig 9: (a) at a distance of LD = 115.36 km, (a) the power profile for the initial pulse in the soliton train, (b) the energy profile for the fundamental soliton train, and (c) the power profile for the third order a soliton train.

6.2. An Example of Single-Channel Soliton Transmission

Therefore, it is imperative that the soliton pulse possess a significant level of strength to ensure the maintenance of the Kerr effect. However, it must also be of a size that is sufficiently small so as to preclude the occurrence of development of solitons of greater order. In this section, To test whether or not the soliton can be used as a reliable information carrier, we will simulate an optical connection with a
single channel. Binary RZ decoder (RZ), soliton pulsed converter (SOLT), and mach zehnder modulation (MZM) are just some of the components that make up this model. optical fibre (OF), compensating fibre (CF), optical amplifier (EDFA), photodiode (PIN), 30 dB gain amplifier (GAIN), low pass filter (FLTR), threshold decision circuit (DIS), random delay (TAU), and finally transmission channel output (OTP). The non-linearity of the fibre introduces the potential of inter-symbol interference, which must be taken into account.

![Fig 10: Different node-level link outputs](image)

a) Refers to the sequence of bits.

b) Denotes the stream of optical carriers in the form of solitons.

c) Relates to the modulation of the soliton stream at the fiber input and modulation of the soliton train at the fiber output.

In order to eradicate the aforementioned phenomenon, The space between two successive solitons provides us with a window of opportunity to choose a pulse intensity that is reasonably modest. To evaluate the viability of using the soliton as a medium for transmission of information, Using these specifics of the individual parts, we will model the connection at a capacity of \( D = 1 \) Gbit/s. Figure 10 displays the simulation results.

The bit sequence that needs to be conveyed, as shown in Figure 10 a, comprising the string "11001010111", functions as a modulator of the soliton stream, which is illustrated in Figure 10.b. The resultant signal emanates as a representation of the transmission of information, as depicted in Figure 10.c. The soliton's presence corresponds to the binary digit "1", while its absence denotes the numerical value of "0" information in binary form is delivered along \( L = 115.36 \) kilometres of optical fibre as a modulated soliton stream. The photodiode then registers the presence of the aforementioned stream.

**CONCLUSION**

In the current research, we have exhaustively explored the evolutionary trajectory of soliton pulses. This particular phenomenon is primarily grounded in the propagation of pulses which is contingent upon linear effects, to wit, Group Velocity Dispersion (GVD), and also non-linear effects which are hinged on the variation of refractive index due to light intensity, otherwise referred to as self phase modulation (SPM). We have successfully demonstrated the fundamental principles of soliton transmission sans distortion, both in terms of shape and speed. Furthermore, we have unequivocally established the fact that the soliton wave is eminently suited for an extensive range of exceedingly high speed transmission applications. It has been evinced through our feasibility study that the potentiality of the interaction between adjacent solitons could be mitigated within the same channel or even adjacent channels under the purview of nonlinear and linear effects (GVD, SPM, XPM, 4WM, etc.). It is noteworthy that the compensating point (SPM + GVD\( \cong 0 \)) is discovered to be efficacious in retaining the soliton wave in the transmission.
References


